Let, for example, the vortices of the odd pairs move initially along the x axis. It is easy to show that in this case $C < C_1$ for any value of ξ. If the disturbance is such that vortices of odd pairs move along the y axis only, then $C > C_1$ irrespective of the value of ε, i.e., purely longitudinal disturbances give rise to the formation of cells in the street, while purely transverse disturbances separate the starting street into two streets.

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A METHOD FOR THE SOLUTION OF NONSTATIONARY PROBLEMS FOR A LAYER OF LIQUID WITH MIXED BOUNDARY CONDITIONS

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In this paper we develop a method for the solution of nonstationary problems with mixed boundary conditions for a layer of heavy liquid. In contrast to well-known analytical–numerical approaches (see [1, 2]), the method we propose makes it possible, being based on a factorization method, to carry out an analytical study of the process of excitation and the establishment of waves.

By way of illustrating, we consider the problem of generating excitations by means of a set of external pressures applied to the upper boundary of a layer of liquid partially covered by an eleastic plate. We model a nonstationary process involving the interaction of waves, excited through baric formations, with a limited ice field.

Mathematically stated, our problem has the form

$$\partial \mathbf{v}/\partial t = -\rho_*^{-1}\nabla p_*, \quad -\infty < x, y < \infty, \ -H \leqslant z \leqslant 0, \ \operatorname{div}\mathbf{v} = 0; \tag{1}$$

$$z = 0, \ p_* = q + \rho_* g\zeta + \begin{cases}\Pi\zeta, & \mathbf{x}\in\Omega,\\ 0, & \mathbf{x}\notin\Omega,\end{cases} \quad w = \partial\zeta/\partial t; \tag{2}$$

$$R = M_n = M_\tau = 0, \ \mathbf{x}\in\partial\Omega, \ \mathbf{x} = \{x, y\}; \tag{3}$$

$$\Pi = d_0\nabla^4 + \rho_0 h\frac{\partial^2}{\partial t^2}, \ q = \begin{cases} q(\mathbf{x}, t), & \mathbf{x}\in D, \ t > 0,\\ 0, & \mathbf{x}\notin D,\end{cases}$$

$$z = -H, \quad w = 0;$$

$$t = 0, \left\{\mathbf{v}, q, \zeta, \frac{\partial\zeta}{\partial t}\right\} = 0. \tag{4}$$

Here {x, y, z} is a rectangular Cartesian coordinate system with origin on the unperturbed free surface of the liquid; the z axis is directed vertically upwards; t is the time; p_* is the dynamic component of the total pressure p in the liquid; v = {u, v, w} is the velocity vector; ρ_* and H are the density and thickness of the layer of liquid; ζ is the elevation of the free surface, coinciding in the domain Ω occupied by the plate with its vertical displacement. R, M_n, and M_τ are the intersecting force, the bending moment, and the torque on the end of the plate $\partial\Omega$; d_0, ρ_0, and h are the stiffness, the density, and the thickness of the plate; q(x, t) is the external perturbing pressure, specified in the domain $D = D_1 \cup D_2$, acting on the free surface of the liquid in the domain D_1 and on the plate in D_2; g is the gravitational acceleration.

We introduce dimensionless variables, identifying them with the subscript 1:

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$$\{x, z, \zeta\} = H\{x_1, z_1, \zeta_1\}, \ v = cv_1,$$
$$c = \sqrt{gh}, \ t = Hc^{-1}t_1, \ \{p, q\} = \rho c^2\{p_1, q_1\},$$
$$d_0 = \rho g H^4 d_1, \ \rho_0 = Hh^{-1}\rho_*\rho_1. \tag{5}$$

Since throughout the sequel only dimensionless variables are employed, we shall omit this subscript henceforth.

Taking the Laplace transform with respect to t and the Fourier transform with respect to x, we reduce the problem (1)-(4) to the integral equation

$$\int_\Omega p_L(\xi, s)\, k(x-\xi, s) = \psi(x, s) + \sum_{j=1}^{2} f_j(x, s), \ x \in \Omega, \ \mathrm{re}\, s \geqslant s^* > 0, \tag{6}$$
$$\{p_L(x, s), q_L(x, s)\} = \int_0^\infty \{p(x, t), q(x, t)\}\, e^{-st} dt,$$
$$f_j(x, s) = \int_{D_j} q_L(x, s)\, k_j(x-\xi, s)\, d\xi,$$
$$\{k(x, s), k_j(x, s)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \{K(\alpha, s), K_j(\alpha, s)\}\, e^{-i(\alpha\cdot x)}\, d\alpha,$$
$$\alpha = \{\alpha_1, \alpha_2\}, \ \alpha = |\alpha|, \ x = \{x, y\},$$
$$K(\alpha, s) = [1 + \rho\varkappa_1^2(\alpha)][s^2 + \varkappa_2^2(\alpha)]\{\rho[s^2 + \varkappa_0^2(\alpha)][s^2 + \varkappa_1^2(\alpha)]\}^{-1},$$
$$K_1(\alpha, s) = -\varkappa_1^2(\alpha)[s^2 + \varkappa_1^2(\alpha)]^{-1}, \ K_2(\alpha, s) = \rho^{-1}[s^2 + \varkappa_0^2(\alpha)]^{-1},$$
$$\varkappa_0^2(\alpha) = \rho^{-1}\alpha^4 d, \ \varkappa_1^2(\alpha) = \alpha\, \mathrm{th}\, \alpha, \ \varkappa_2^2(\alpha) = \varkappa_1^2(\alpha)(1 + \alpha^4 d)[1 + \rho\varkappa_1^2(\alpha)],$$

where $\psi(x, s)$ is the Laplace transform of the general solution of the equation $\Pi\psi = 0$. In this we understand that if Ω is an unbounded domain, then $\psi(x, s)$ will be the solution which satisfies radiation conditions for the waves at infinity. In Eq. (6) the quantity s^* is the abscissa of convergence of the Laplace transform.

If the plate occupies the halfplane (Ω: $x \geq 0$) and if the wave picture is identical at an arbitrary cross-section y = const (the two-dimensional case), it follows that in the formula (6) the vectors x and ξ should be replaced by the scalar variables x and ξ. In this regard, we write the function $\psi(x, s)$ in the form

$$\psi(x, s) = \begin{cases} C_1(s)\, e^{-\lambda x} + C_2(s)\, e^{i\lambda x}, & x \geqslant 0, \\ 0, & x < 0, \end{cases} \tag{7}$$
$$C_1(s) = (4d\lambda^3)^{-1}[(1-i)\sigma_+(\lambda, s) - i\sigma_+(i\lambda, s)],$$
$$C_2(s) = (4d\lambda^3)^{-1}[\sigma_+(\lambda, s) + (1-i)\sigma_+(i\lambda, s)],$$
$$\sigma_+(\alpha, s) = P_+(\alpha, s) - Q_2(\alpha, s),$$
$$\{P_+(\alpha, s), Q_2(\alpha, s)\} = \int_\Omega \{p_L(x, s), q_L(x, s)\}\, e^{i\alpha x} dx,$$
$$\lambda \equiv \lambda(s) = (-\rho s^2 d^{-1})^{1/4}, \ 0 \leqslant \arg\lambda(s) < 0{,}5\,\pi.$$

Here the coefficients C_1 and C_2 are determined by requiring the solution for the plate to satisfy the boundary conditions (3) on $\partial\Omega(x = 0, \ \partial^2\zeta/\partial x^2 = \partial^3\zeta/\partial x^3 = 0)$.

By extending Eq. (6) into the region outside of Ω (on the negative semiaxis $x < 0$) by means of the function $\phi(x, s)$ and applying a Fourier transform with respect to the variable x, we reduce the problem to the equivalent functional equation

$$P_+(\alpha, s)\, K(\alpha, s) + \Phi_-(\alpha, s) = \Psi_+(\alpha, s) + \sum_{j=1}^{2} Q_j(\alpha, s)\, K_j(\alpha, s), \tag{8}$$

$$\alpha \in E, \operatorname{re} s \geqslant s^* > 0, \; Q_j(\alpha, s) = \int_{D_j} q_L(x, s)\, e^{i\alpha x} dx, \; D_1\colon x \in (-\infty, 0],$$
$$D_2\colon x \in [0, \infty),$$
$$\Psi_+(\alpha, s) = \int_0^\infty \psi(x, s)\, e^{i\alpha x} dx, \; \Phi_-(\alpha, s) = \int_{-\infty}^0 \varphi(x, s)\, e^{i\alpha x} dx.$$

We now investigate the singularities (zeros and poles) of the functions $K(\alpha, s)$, $K_j(\alpha, s)$. The equation $s^2 + \kappa_j^2(\alpha) = 0$ ($j = 1, 2$) has for $s = -i\omega$ ($-\infty < \omega < \infty$) the two real roots (see [3, 4]) $\alpha = \pm z_j(-i\omega)$ and a denumerable set of complex roots $\alpha = \pm i\eta_{j,m}(-i\omega)$, $m = 1, 2, 3, \ldots$ (for $j = 1$, they are imaginary). The equation $s^2 + \kappa^2(\alpha) = 0$ is satisfied by the two real roots $\alpha = \pm\lambda(-i\omega)$ and the two imaginary roots $\alpha = \pm i\lambda(-i\omega)$, where $\lambda(s)$ is given in the relations (7).

If $\operatorname{re} s > 0$, the singular points λ, z_j are shifted from the real axis into the complex plane. In this connection, the choice of sign for the root in $\kappa_j(\alpha)$, coinciding with the sign of the variable α when its values are real, stipulate the fixing of the roots $\alpha = \lambda(s)$, $\alpha = z_j(s)$ in the upper halfplane for α. The remaining singularities are complex and do not appear on the real axis.

Thus the functional equation (8) has, for the functions appearing in it, a common strip E of regularity, which contains the whole real axis for α, i.e.,

$$E\colon -\infty < \operatorname{re} \alpha < \infty, \; \gamma_- < \operatorname{im} \alpha < \gamma_+, \; \gamma_- < 0 < \gamma_+.$$

Throughout the relations (8) and in what follows, the subscripts + or − indicate regularity of the function with respect to the variable α in the upper ($\operatorname{im} \alpha > \gamma_-$) and lower ($\operatorname{im} \alpha < \gamma_+$) halfplanes.

Factorization (see [5, 6]) of Eq. (8) enables us to represent the solution in integral form. Thus we find the following for the pressure p under the plate and the displacement ζ of the upper boundary of the layer of liquid:

$$p(x, t) = \frac{1}{4\pi^2 i} \int_{-\infty}^{\infty} e^{-i\alpha x} \int_{\delta - i\infty}^{\delta + i\infty} [Q_2(\alpha, s) + G_+(\alpha, s) K_+^{-1}(\alpha, s)]\, e^{st} ds\, d\alpha, \; x \geqslant 0; \quad (9)$$

$$\zeta(x, t) = \frac{1}{4\pi^2 i} \int_{-\infty}^{\infty} e^{-i\alpha x} \int_{\delta - i\infty}^{\delta + i\infty} Z(\alpha, s)\, e^{st} ds\, d\alpha, \; -\infty < x < \infty, \quad (10)$$
$$K(\alpha, s) = K_+(\alpha, s) K_-(\alpha, s),$$
$$Z(\alpha, s) = K_1(\alpha, s) \left[\sum_{j=1}^{2} Q_j(\alpha, s) + G_+(\alpha, s) K_+^{-1}(\alpha, s)\right], \; \delta \geqslant s^* > 0.$$

In the expressions (9) and (10) the function $G_+(\alpha, s)$ has the form

$$G_+(\alpha, s) = \frac{1}{2\pi i} \int_{\Gamma_+} \sum_{j=1}^{2} Q_j(\xi, s) K_1(\xi, s) K_-^{-1}(\xi, s)\, \chi(\xi, \alpha, s)\, d\xi, \quad (11)$$
$$\chi(\xi, \alpha, s) = \frac{1}{\xi - \alpha} + \frac{i}{4\lambda^3 \Delta(s)\, d} \left[\frac{\Delta_1(\xi, s)}{(\alpha + i\lambda) K_+(i\lambda, s)} + \frac{\Delta_2(\xi, s)}{(\alpha + \lambda) K_+(\lambda, s)}\right],$$
$$\Delta_1(\xi, s) = \frac{(1 - i)\, K_+(i\lambda, s)}{\xi - \lambda} - \frac{iK_+(\lambda, s)}{\xi - i\lambda} + \frac{i(\xi + \lambda)(\xi - \lambda)^{-1}(\xi - i\lambda)^{-1}}{K_+(\lambda, s)\, 8\lambda^4 d},$$
$$\Delta_2(\xi, s) = \frac{K_+(i\lambda, s)}{\xi - \lambda} + \frac{(1 - i)\, K_+(\lambda, s)}{\xi - i\lambda} - \frac{(\xi + i\lambda)(\xi - \lambda)^{-1}(\xi - i\lambda)^{-1}}{K_+(i\lambda, s)\, 8\lambda^4 d},$$
$$\Delta(s) = K_+(\lambda, s) K_+(i\lambda, s) + [K_+(\lambda, s) K_+(i\lambda, s)\, 64\lambda^8 d^2]^{-1} -$$

$$- i(8\lambda^4 d)^{-1}\left[K_+(i\lambda, s)K_+^{-1}(\lambda, s) - K_+(\lambda, s)K_+^{-1}(i\lambda, s) - 4i\right]. \quad (11)$$

From the requirement that $G_+(\alpha, s)$ be represented by an integral of Cauchy type, it follows that the contour Γ_+ is located in the strip of regularity E below the real axis for α. Thus, in the planar case, the relations (9)-(11) furnish the exact solution of the problem (1)-(4).

We now consider an example in which the external load acting on the surface of the plate obeys, for $t > 0$, the following harmonic law:

$$q(x, t) = \begin{cases} f(x)e^{-i\omega t}, & t > 0,\ x \geqslant 0,\ \omega > 0, \\ 0, & t \leqslant 0. \end{cases} \quad (12)$$

We assume now that the function f(x) satisfies conditions for the existence of the Fourier integral. Into Eq. (11) we substitute the $Q_j(\alpha, s)$ corresponding to the function (12); we then close the contour Γ_+ in the upper or lower halfplane and, proceeding from the condition for decrease of exponential functions in the integrand, we then evaluate $G_+(\alpha, s)$ by residues. Next we handle the inner integrals in the solutions (9) and (10), taking into account the position of the poles $s_0 = -i\omega$, $s_1 = -i\kappa_1(\alpha)$, $s_2 = i\kappa_2(\alpha)$ on the imaginary axis, on the basis of Jordan's Lemma.

For the outer integrals in the solutions (9) and (10) we go to an integration contour Γ, which runs along the real axis, making a small semicircular detour around the point $\alpha = -z_2(-i\omega)$ in the upper half-plane and a similar detour around the point $\alpha = z\,(-i\omega)$ in the lower half-plane. The contour Γ, chosen in accordance with the limiting amplitude principle (see [3]), makes it possible to carry out a term-by-term integration:

$$\zeta(x, t) = e^{-i\omega t} I_0(x) + \sum_{j=1}^{2} I_j(x, t), \quad (13)$$

$$I_0(x) = \frac{1}{2\pi}\int_\Gamma B_0(\alpha)e^{-i\alpha x}d\alpha,\ I_j(x, t) = \frac{1}{2\pi}\int_\Gamma B_j(\alpha)e^{-ix\varphi_j(\alpha,\gamma)}d\alpha,$$

$$\varphi_j(\alpha, \gamma) = \alpha + (-1)^{j+1}\gamma\varkappa_j(\alpha),\ \gamma = t/x,$$

$$B_0(\alpha) = F(\alpha)K_3(\alpha, -i\omega) + K_1(\alpha, -i\omega)K_+^{-1}(\alpha, -i\omega)N(\alpha, -i\omega),$$

$$B_1(\alpha) = \varkappa_1(\alpha)N(\alpha, -i\varkappa_1(\alpha))\{2K_+(\alpha, -i\omega)[\varkappa_1(\alpha) - \omega]\}^{-1},$$

$$B_2(\alpha) = \frac{\varkappa_1^2(\alpha)F(\alpha) - K_-(\alpha, i\varkappa_2(\alpha)N(\alpha, i\varkappa_2(\alpha))\left[\varkappa_1^2(\alpha) - \varkappa_2^2(\alpha)\right]}{2\varkappa_2(\alpha)\left[1 + \rho\varkappa_1^2(\alpha)\right][\omega + \varkappa_2(\alpha)]},$$

$$K_3(\alpha, s) = \frac{K_1(\alpha, s)K_2(\alpha, s)}{K(\alpha, s)},$$

$$N(\alpha, s) = \sum_{n=0}^{\infty}\frac{F(\alpha_n)K_1(\alpha_n, s)}{K'_-(\alpha_n, s)}\chi(\alpha_n, \alpha, s),$$

$$F(\alpha) = \int_0^\infty f(x)e^{i\alpha x}dx,\ K'_-(\alpha, s) = \frac{\partial}{\partial\alpha}K_-(\alpha, s),$$

$$\alpha_0 = z_2(s),\ \alpha_n = i\eta_{2,n}(s),\ \varkappa_2^2(\alpha_n) + s^2 = 0.$$

In Eq. (13) the first term determines the stationary part of the solution, the second and third terms give the nonstationary contribution.

In Eq. (12) we assume now that the external load is distributed according to the law

$$f(x) = \begin{cases} e^{i\eta(x-a_1)}, & x \in [a_1, a_2], \\ 0, & x \notin [a_1, a_2],\ a_1, a_2 > 0. \end{cases}$$

In Eq. (13) we can then put

$$F(\alpha) = F_1(\alpha) + F_2(\alpha),\ F_j(\alpha) = \frac{(-1)^j e^{-i\eta a_1}}{i(\eta + \alpha)}e^{i(\eta+\alpha)a_j}. \quad (14)$$

We evaluate $I_0(x)$ with the aid of residue theory. We single out the principal parts $I_j(x, t)$ by deforming the contour Γ close to the poles. Estimating the remainders by the stationary phase method (see [6, 7]), we obtain the elevation ζ of the free surface of the liquid in different space-time domains. For example, for $a_2c_2^{-1} < t < (a_1 + a_2)c_2^{-1}$, on the free surface of the liquid (x < 0) we have

$$\begin{aligned}
&\zeta = O(|x|^{-\nu}),\ x < -c_1t, \\
&\zeta = \zeta_{-3} + O(|x|^{-\nu}),\ -c_1t < x < -c_1(t - a_1c_2^{-1}), \\
&\zeta = \zeta_{-1} + \zeta_{-3} + O(|x|^{-\nu}),\ -c_1(t - a_1c_2^{-1}) < x < -c_1(t - a_2c_2^{-1}), \\
&\zeta = \sum_{j=1}^{3} \zeta_{-j} + O(t^{-1}),\ -c_1(t - a_2c_2^{-1}) < x \ll 0.
\end{aligned} \tag{15}$$

The oscillations of the plate (x > 0) for these times are described by analogous expressions:

$$\begin{aligned}
&\zeta = \sum_{j=1}^{5} \zeta_j + \theta,\ 0 < x < -a_2 + c_2t, \\
&\zeta = \sum_{j=1}^{4} \zeta_j + \theta,\ -a_2 + c_2t < x < -a_1 + c_2t, \\
&\zeta = \sum_{j=1}^{3} \zeta_j + \theta,\ -a_1 + c_2t < x < a_1, \\
&\zeta = \sum_{j} \zeta_j + \theta\ (j = 0, 2, 3, 6),\ a_1 < x < a_2, \\
&\zeta = \sum_{j} \zeta_j + \theta\ (j = 3, 6, 7),\ a_2 < x < c_2t, \\
&\zeta = \zeta_6 + \zeta_7 + O(|x - a_2|^{-\nu}),\ c_2t < x < a_1 + c_2t, \\
&\zeta = \zeta_7 + O(|x - a_2|^{-\nu}),\ a_1 + c_2t < x < a_2 + c_2t, \\
&\zeta = O(|x - a_2|^{-\nu}),\ x > a_2 + c_2t,
\end{aligned} \tag{16}$$

$$c_m = \varkappa_m'(z_m),\ z_m = z_m(-i\omega),\ \varkappa_m^2(z_m) - \omega^2 = 0,\ m = 1, 2.$$

Here

$$\begin{aligned}
&\zeta_{-j} = -\frac{i\omega}{2c_1}\frac{N_j(z_1)}{K_+(z_1)} e^{-i(z_1x+\omega t)},\ j = 1, 2, 3, \\
&\zeta_0 = \frac{-\varkappa_1^2(\eta)}{[1 + \rho\varkappa_1^2(\eta)][\varkappa_2^2(\eta) - \omega^2]} e^{i[\eta(x-a_1)-\omega t]}, \\
&\zeta_j = \frac{-i\varkappa_2^2(z_2)F_j(z_2)}{2\omega c_2[1 + \rho\varkappa_1^2(z_2)]} e^{-i(z_2x+\omega t)},\ j = 1, 2, \\
&\zeta_j = \frac{-i\varkappa_1^2(z_2)(z_2^4d - \rho\omega^2)}{2\,[1 + \rho\varkappa_1^2(z_2)]} K_+(z_2, -i\omega) N_{j-2}(-z_2) e^{i(z_2x-\omega t)},\ j = 3, 4, 5, \\
&\zeta_j = \frac{-i\varkappa_2^2(z_2)F_{j-5}(-z_2)}{2\omega c_2[1 + \rho\varkappa_1^2(z_2)]} e^{i(z_2x-\omega t)},\ j = 6, 7, \\
&N_j(\alpha) = \frac{F_j(z_2)K_1(z_2, -i\omega)}{K_-'(z_2, -i\omega)}\chi(z_2, \alpha, -i\omega),\ j = 1, 2, \\
&N_3(\alpha) = \sum_{n=1}^{\infty} \frac{F(\alpha_n)K_1(\alpha_n, -i\omega)}{K_-'(\alpha_n, -i\omega)}\chi(\alpha_n, \alpha, -i\omega), \\
&\alpha_n = i\eta_{2,n},\ \varkappa_2^2(\alpha_n) - \omega^2 = 0, \\
&\theta = O(|x|^{-\nu}) + \begin{cases} \sum_{j=1}^{2} O(|x - a_j|^{-\mu}), & |x - a_j| \to \infty, \\ O(t^{-1}),\ t \to \infty, & |x - a_j| \leqslant \text{const}, \end{cases} \\
&1/3 \leqslant \nu,\ \mu \leqslant 1,\ |\gamma| > 0.
\end{aligned} \tag{17}$$

Here $K(\alpha, -i\omega)$, $K_1(\alpha, -i\omega)$, $\kappa_1(\alpha)$, $\kappa_2(\alpha)$ are defined in Eqs. (6), $\chi(\xi, \alpha, s)$ in Eq. (11), and $F_j(\alpha)$ in Eq. (14).

An analysis of the solution (15)-(17) shows that each end of the interval $[a_1, a_2]$ to which a perturbation of the pressure $q(x, t)$ is applied radiates waves in both directions; these waves do not attenuate with distance and propagate with group velocity c_1 in the liquid and with velocity c_2 in the plate. The waves ζ_6, ζ_1 are generated by the edge

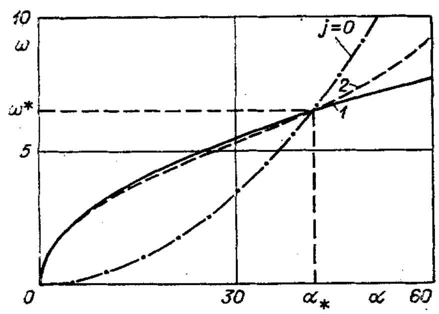

Fig. 1

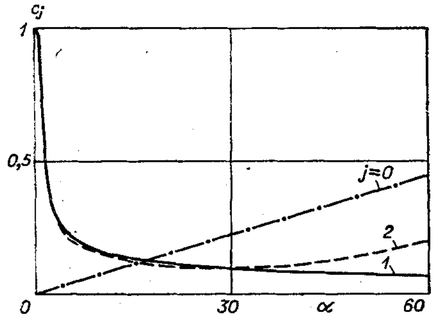

Fig. 2

$z = a_1$, while the waves ζ_7, ζ_2 are generated at the point a_2, to the right and to the left, respectively. Starting from the initial time instant the edge of the plate $x = 0$ is the source of the waves ζ_{-3} in the liquid and ζ_3 in the plate with amplitude of the order $\exp(-\beta a_1)$, where $\beta = \inf|\mathrm{re}\,\eta_{2,n}|$, $n \geqslant 1$.

To the waves ζ_1, ζ_2 there correspond the waves ζ_4, ζ_5, reflected from the plate boundary, and the waves ζ_{-1}, ζ_{-2}, refracted into the liquid. The contribution ζ_0 arises from the specific form of the external pressure function $f(x)$.

A comparative analysis of the dispersion functions $\omega = \kappa_j(\alpha)$, shown in Fig. 1 for a layer of liquid covered by a plate ($j = 2$, then separately for the plate itself ($j = 0$), and, finally, for a layer of liquid with a free surface ($j = 1$), shows that for low frequencies of the oscillations the plate changes the lengths of the surface waves insignificantly. For high frequencies ($\omega \gg \omega^*$) the influence of the liquid on the length of the waves perturbed under the plate is small. This is a consequence of the coincidence in the asymptotic behavior of the dispersion curves $\omega = \kappa_j(\alpha)$ ($j = 0, 2$) as $\alpha \to \infty$. In the intermediate frequency range the dispersion of the gravitational–elastic waves studied is different from the limiting behavior in the $j = 0$ and $j = 1$ situations. For $\omega = \omega^*$ the curves intersect; the waves generated are all of the same length $\ell = 2\pi/\alpha_*$.

Figure 2 reflects the behavior of the group velocities in the cases for $c_j = \kappa_j'(\alpha)$, as indicated. It is evident that the plate slows the propagation of the long waves on the surface of the liquid. In Figs. 1 and 2 the values of the dimensionless parameters are: $d = 6\cdot 10^{-8}$, $\rho = 4.32\cdot 10^{-2}$, $\alpha_* = 41.69$.

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